# Why We Can Approximate Spheroidal Geopotential Surfaces as Spherical but Can't Approximate True Geopotential Surfaces as Spheroidal

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## ABSTRACT

Meteorologists and oceanographers use spherical, spheroidal, true geopotentials and associated coordinates to represent Earth gravity. The spherical geopotential ($\Phi_s$) coordinate system $[\lambda, \varphi, r]^s$ associated with the standard gravity $\mathbf{g}_s$ is for the Earth with uniform mass density and without rotation. Here, $\lambda$ is the longitude, $\varphi$ the geocentric latitude, $r$ the radial distance. The spheroidal geopotential ($\Phi_a$) coordinate system $[\lambda, \hat{\varphi}, h]^a$ associated with the apparent gravity $\mathbf{g}_a$ is for the Earth with uniform mass density and rotation. Here, $\hat{\varphi}$ the geodetic latitude, $h$ the spheroidal (ellipsoidal) height. The true geopotential ($\Phi_t$) coordinate system $[\lambda, \hat{\varphi}, h_t]^t$ associated with the true gravity $\mathbf{g}_t$ for the Earth with nonuniform mass density and rotation. Here, $h_t$ is the orthometric height. The spherical geopotential is used in almost all atmospheric and oceanic dynamics and models after employing two approximations: (1) spheroidal geopotential approximation (EGA) which is to approximate the true geopotential surfaces as spheroidal, and (2) spherical geopotential approximation (SGA) which is to approximate the spheroidal geopotential surfaces as spherical. The two approximations involve advective metric terms and horizontal (i.e., on geopotential surfaces) pressure gradient errors. The advective metric terms are negligible in all the geopotential coordinates. The horizontal pressure gradient force error is negligible in the SGA but equals the horizontal gravitational disturbance vector ($g_0\nabla N$) in the EGA with $N$ the geoid height. It is urgent to include $g_0\nabla N$ in atmospheric and oceanic dynamics and models if spherical or spheroidal geopotential coordinates are still used.